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The influence of cross-order terms in interface mobilities for structure-borne sound source characterization: Frame-like structures

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Abstract

The applicability of interface mobilities for structure-borne sound source characterization critically depends on the admissibility of neglecting the cross-order terms. Following on from a previous study [H.A. Bonhoff, B.A.T. Petersson, *Journal of Sound and Vibration* 311 (2008) 473–484], the influence of the cross-order terms is investigated for frame-like structures under the assumption of a uniform force-order distribution. Considering the complex power, the cross-order terms are significant from intermediate frequencies on upwards. At lower frequencies, the cross-order terms can come into play for cases where the in-phase motion of the structure along the interface is constrained. The frequency characteristics of the influence of cross-order terms for the zero-order source descriptor and coupling function are similar to those of the complex power. For non-zero source descriptor and coupling function orders, the quality of the equal-order approximation mainly depends on the presence of low-order cross-order interface mobilities. By analyzing the symmetry of an interface system, it is possible to predict which cross-order terms are equal to zero. The equal-order approximation manages to capture the main trends and overall characteristics and offers an acceptable estimate for engineering practice.

1. Introduction

In previously published work [1], the applicability of the concept of interface mobilities was investigated for structure-borne sound source characterization involving plate-like structures. A pivotal factor for the applicability of interface mobilities for source characterization is the admissibility of neglecting the cross-order terms. The present contribution addresses the influence of cross-order terms in interface mobilities for frame-like structures, see Fig. 1. The theoretical background and the fundamental definitions as well as the field of application associated with the concept of interface mobilities for the present study are the same as those presented in Ref. [1].

For the investigation of the influence of cross-order terms and focusing on the interface mobilities, a uniform distribution of force orders can be assumed. Under the assumption of a uniform force-order

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Nomenclature

Symbols

C	interface circumference
F	force distribution
$\hat{F}$	force amplitude
$\hat{F}_q$	force order
k_B	bending wave number
k_p, k_q	interface numbers
L_0	half the diagonal of the rectangular interface
p, q	order numbers
Q	complex power
s	interface coordinate
v	velocity
$\hat{v}_p$	velocity order
Y	mobility
$\overline{Y}(s\|s)$	mean point mobility
$\hat{Y}_{pq}$	interface mobility
$\hat{Y}_{p-p}$	equal-order interface mobility
δ	Dirac delta function
$\Delta\phi$	phase difference between two cross-order terms

Indices

0	excitation
int	integration

distribution, see Section 2, the cross-order terms reduce to the cross-order interface mobilities,

$$\hat{Y}_{pq} = \frac{1}{C^2}\int_0^C\int_0^C Y(s|s_0)\mathrm{e}^{-\mathrm{j}k_p s}\mathrm{e}^{-\mathrm{j}k_q s_0}\,\mathrm{d}s\,\mathrm{d}s_0, \quad p \neq -q \tag{1}$$

with $p, q \in \mathbb{Z}$ and

$$Y(s|s_0) = \sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\hat{Y}_{pq}\mathrm{e}^{\mathrm{j}k_p s}\mathrm{e}^{\mathrm{j}k_q s_0}, \quad k_p = \frac{2p\pi}{C}, \; k_q = \frac{2q\pi}{C}. \tag{2}$$

A list of symbols is given in the nomenclature above. Cross-order interface mobilities describe the sensitivity of point and transfer mobilities to the location along the interface. Ordinary mobilities herein are defined by the distance between excitation and response positions along the interface. Varying the location of a specific point or transfer mobility, therefore, means a change of both excitation and response positions under the condition that $|s - s_0| = \text{const}$. When averaging an ordinary mobility with excitation at s_0 and response position at s over all possible locations along the interface, the dependence on such locations drops out. In the following equation, therefore, only equal-order interface mobilities remain.

$$\frac{1}{C}\int_0^C Y(s_{\mathrm{int}} + s|s_{\mathrm{int}} + s_0)\,\mathrm{d}s_{\mathrm{int}} = \sum_{p=-\infty}^{\infty}\hat{Y}_{p-p}\mathrm{e}^{\mathrm{j}k_p(s+s_0)} \tag{3}$$

Frame-like structures often exhibit some kind of symmetry, e.g. geometrical symmetry. The kind of symmetry relevant for interface mobilities is established when the dynamic characteristics of the structure are equal in both directions along the interface outgoing from a certain point. Such points can be termed symmetry points. An example of a symmetric interface system is presented in Fig. 1.

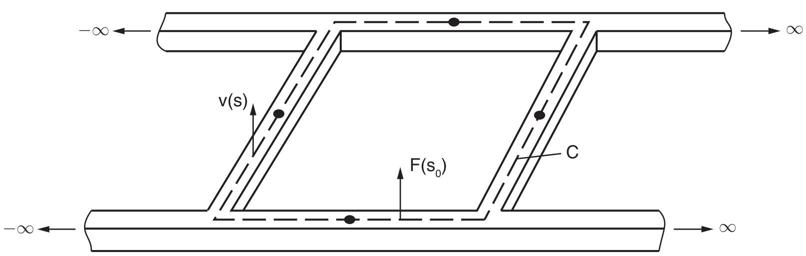

Fig. 1. Illustration of an interface located on a frame-like structure constructed of two pairs of parallel beams of equal material and cross-section. – – –, Interface; •, symmetry point.

For asymmetric interface systems, the results obtained are mainly dependent on the structural dynamic properties of the systems. In such cases, the results can be interpreted without considering the forces. For symmetric interface systems, however, the force distributions have to be included in the analysis.

In order to shed some light on the above statements regarding the importance of symmetry, the assumption of a uniform force-order distribution and its implications will be discussed in detail in the following two sections. It shall be pointed out here that Sections 3 and 4 are generally applicable, although exemplified through cases of frame-like structures. In Section 4, experimental and analytical results are presented regarding the influence of cross-order terms in interface mobilities for frame-like structures.

2. Uniform force-order distribution

A spatial force distribution $F(s_0)$ with an ideal point excitation at $s_0 = 0$ results in uniformly weighted force orders. Such a force distribution can be written as $F(s_0) = \hat{F}\delta(s_0)$, where $\delta(s_0)$ is the Dirac delta function. Hence, the origin of the interface coordinates s and s_0 is set to the location of the ideal point excitation. The force orders now follow:

$$\hat{F}_q = \frac{1}{C}\int_0^C \hat{F}\delta(s_0)\mathrm{e}^{-\mathrm{j}k_q s_0}\,\mathrm{d}s_0 = \hat{F}. \quad (4)$$

For the investigation of the influence of the cross-order terms under the assumption of a uniform force-order distribution, the force orders can be substituted by $\hat{F}$.

In the derivation of the source descriptor and coupling function orders, the cross-order terms have to be omitted already in the following equation, see Ref. [1]:

$$\hat{v}_p = C\sum_{q=-\infty}^{\infty} \hat{Y}_{pq}\hat{F}_{-q}. \quad (5)$$

The influence of the cross-order terms for the source descriptor and coupling function orders can therefore be assessed by analyzing the velocity orders. Under the assumption of a uniform force-order distribution, the velocity orders can be written as

$$\hat{v}_p = C\hat{F}\sum_{q=-\infty}^{\infty} \hat{Y}_{pq} \approx C\hat{F}\hat{Y}_{p-p}. \quad (6)$$

By neglecting the cross-order interface mobilities, the equal-order approximation is gained, as shown on the right-hand side of Eq. (6).

In contrast, the complex power can be established without neglecting the cross-order terms [1].

$$Q = \frac{C^2}{2}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty} \hat{F}_p^*\hat{Y}_{pq}\hat{F}_{-q} \quad (7)$$

Hence, the complex power consists of a series of cross- and equal-order terms different from those of the source descriptor and coupling function orders. By assuming a uniform force-order distribution, the complex power becomes proportional to the sum of all interface mobility terms in the strict formulation. When omitting the cross-order terms it is proportional to the superposition of all equal-order interface mobilities only:

$$Q = \frac{C^2\hat{F}^2}{2}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty} \hat{Y}_{pq} \approx \frac{C^2\hat{F}^2}{2}\sum_{p=-\infty}^{\infty} \hat{Y}_{p-p}. \quad (8)$$

Henceforth, the influence of the cross-order terms with respect to the complex power can be analyzed by comparing the sum of all interface mobilities with the sum of all equal-order interface mobilities. As shown in Ref. [1], the superposition of all interface mobilities equals the point mobility at the origin of the interface coordinates, where $s = s_0 = 0$, i.e.,

$$Y(0|0) = \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \hat{Y}_{pq}. \tag{9}$$

Furthermore, the sum of all equal-order interface mobilities equals the mean point mobility along the interface, [1], i.e.

$$\overline{Y}(s|s) = \frac{1}{C}\int_0^C Y(s|s)\,\mathrm{d}s = \sum_{p=-\infty}^{\infty} \hat{Y}_{p-p}. \tag{10}$$

In turn, the sum of all cross-order interface mobilities equals the difference between $Y(0|0)$ and $\overline{Y}(s|s)$.

Eqs. (9) and (10) lead to substantial simplifications for the analysis of the influence of the cross-order terms with respect to the complex power. Only point mobilities have to be measured or calculated. The assessment of the cross-order term influence for the individual source descriptor and coupling function orders, however, requires the individual interface mobility terms. Hence, the complete point and transfer mobility matrix has to be measured or calculated.

As seen in Eqs. (6) and (8), only mobilities remain to be compared in order to assess the importance of the cross-order terms. Hence, the analysis is dependent on the structural dynamic characteristics of the system under consideration only. For a different location of the ideal point excitation, however, the sum of all interface mobilities will change, see Eq. (9). This is due to the fact that for a uniform force-order distribution, the origin of the interface coordinates has to coincide with the position of the point force. In a strict sense, the results for an interface system with a given spatial force distribution are representative for that particular combination of physical system and force distribution only.

The difference between Eqs. (9) and (10), which will vary for all possible locations of the ideal point excitation, equals the sum of all cross-order interface mobilities. Such variations result from changes in the constructive and destructive interference of the cross-order interface mobilities when superimposed. In conjunction with the previously cited work on plate-like structures, it is observed that these variations are of minor significance for asymmetric interface systems and therefore can be neglected. Hence, the results for the cross-order term influence for a given asymmetric system are approximately independent of the spatial force distribution. The overall characteristics for a single position of the ideal point excitation, therefore, are representative for all other possible locations.

For symmetric interface systems, however, large variations can be expected between the sums of cross-order interface mobilities for different locations of the ideal point excitation. One specific case of a uniform force-order distribution, therefore, no longer is representative for the system under consideration in general. Hence, the location of the ideal point excitation has to be varied in order to obtain representative results for the influence of the cross-order terms on a symmetric interface system under the assumption of a uniform force-order distribution.

3. Interference of cross-order interface mobilities

As pointed out in the preceding section, the superposition of cross-order interface mobilities is sensitive to the location of the ideal point excitation. Furthermore, such variation in constructive and destructive interference between the cross-order interface mobilities is observed to be more distinct for symmetric interface systems than for asymmetric ones. In the present section, the interference process of cross-order interface mobilities is investigated in order to gain insight into the consequences of the two statements above.

The presence of symmetry points on interface systems originates from axis symmetry of the dynamic characteristics of the structure. Such an axis crosses the interface at two points, see Fig. 1. Hence, only an even number of symmetry points is possible. Furthermore, the symmetry points have to be distributed with equal distance along the interface.

The dynamic characteristics of a structure along a given interface are independent of the spatial force distribution and the coordinate system. Hence, the net influence of the individual interface mobility terms, i.e.

their magnitude will be constant for all possible locations of the origin of the interface coordinates. However, when changing the point where $s = s_0 = 0$, the phase of the cross-order interface mobilities will change.

In principle, constructive or destructive interference is possible for any combination of cross-order interface mobilities, provided they are not equal to zero. In Fig. 2, two examples are illustrated. At random, two cross-order interface mobilities are combined. The phase difference between the two cross-order interface mobilities is plotted as a function of the location of origin of the interface coordinates. At a phase difference of $\Delta\Phi = \pi$, the two cross-order interface mobilities will interfere destructively. If the origin of the interface coordinates is located at a point corresponding to a zero phase shift, the two cross-order interface mobilities under consideration will add up. In the majority of cases, however, the cross-order interface mobilities will not have the same magnitude. Also, the phase relations shown in Fig. 2 will vary with frequency. Thus, a distinct constructive or destructive interference at all frequencies is rather unlikely. However, for symmetric interface systems, cross-order interface mobility combinations of type $\hat{Y}_{pq}$ and $\hat{Y}_{-p-q}$ can add up or cancel completely throughout the entire frequency range. Such combinations of cross-order interface mobilities are henceforth referred to as cross-order interface mobility pairs.

The two interface mobility terms in a cross-order interface mobility pair are characterized by equal magnitude but opposing sign of the order numbers. The magnitude of an order number describes the number of wavelengths along the interface of the velocity or force order under consideration. The sign of an order number stands for the direction along the interface of the corresponding order. Hence, the two cross-order interface mobilities describe the coupling between the same velocity and force orders but with opposite direction along the interface.

If the origin of the interface coordinates is located at a symmetry point, the dynamic properties of the structure are equal in both directions along the interface. Consequently, the two interface mobilities forming a cross-order interface mobility pair are identical. Hence, a symmetry point designates a point of origin of the interface coordinates where the two terms in all cross-order interface mobility pairs are in-phase and therefore

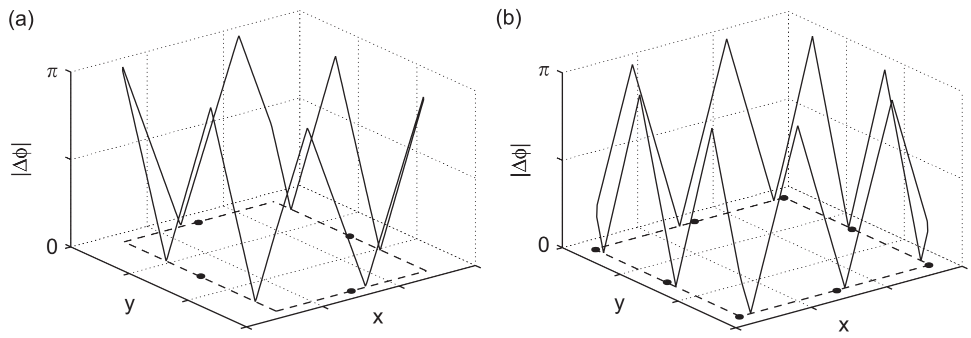

Fig. 2. Two examples of the phase difference between two cross-order interface mobilities as a function of the location of origin of the interface coordinates: (a) $\hat{Y}_{0-2}$ and $\hat{Y}_{22}$; (b) $\hat{Y}_{13}$ and $\hat{Y}_{-62}$. – – –, Interface; —, phase difference; •, symmetry point.

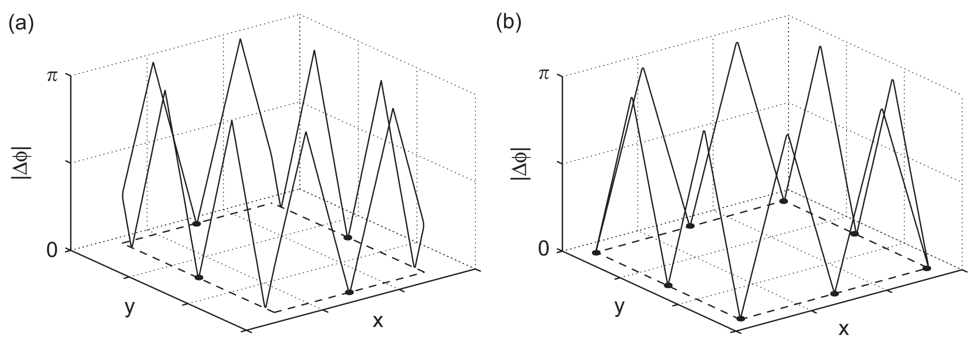

Fig. 3. Two examples of the phase difference within cross-order interface mobility pairs as a function of the location of origin of the interface coordinates: (a) $\hat{Y}_{22}$ and $\hat{Y}_{-2-2}$; (b) $\hat{Y}_{04}$ and $\hat{Y}_{0-4}$. – – –, Interface; —, phase difference; •, symmetry point.

Table 1
Cross-order interface mobilities which are equal to zero due to symmetry

Number of symmetry points	Cross-order terms that are zero
2	—
4	$\hat{Y}_{pq} = 0$ for $\|p+q\|$ odd
6	$\hat{Y}_{pq} = 0$ for $\|p+q\| = 1$ or even
8	$\hat{Y}_{pq} = 0$ for $\|p+q\| = 2$ or odd

add up when superimposed, see Fig. 3. The distance along the interface between points of origin relating to phase differences of π and 0 is found to be given by

$$\Delta s = \frac{C}{4|p+q|} \quad \text{for } \hat{Y}_{pq} \text{ and } \hat{Y}_{-p-q} \text{ with } p+q \neq 0. \tag{11}$$

For certain cross-order interface mobility pairs on systems with more than two symmetry points, not all symmetry points will coincide with points of zero phase difference. At each symmetry point, the two interface mobilities in every cross-order interface mobility pair have to be in phase. However, the latter is not possible if no integer multiple of $2\Delta s$, see Eq. (11), equals the distance along the interface between two points of symmetry. As the result, the two terms in a cross-order interface mobility pair for which the symmetry points will not coincide with points of zero phase difference are equal to zero. Thus, the coupling between the corresponding excitation and response orders is not possible due to symmetry.

On an interface system with four points of symmetry for instance, see Fig. 1, a coupling between first-order excitation and zero-order response is not possible. This is due to the fact that the terms $\hat{Y}_{01}$ and $\hat{Y}_{0-1}$ feature only two points along the interface with a zero phase difference, see Eq. (11). Furthermore, all cross-order interface mobilities with an odd number of $p+q$ are equal to zero on interface systems with four symmetry points. A list of cross-order interface mobilities which are equal to zero due to symmetry is given in Table 1.

As the cross-order interface mobility pairs cover all cross-order interface mobilities, every term has its counterpart. Complete constructive interference within all pairs occurs when the location of origin of the interface coordinates coincides with a symmetry point. The superposition of all cross-order interface mobilities in that case does not necessarily constitute a maximum, since the cross-order interface mobility pairs interfere as well. Neither is a complete destructive interference within all pairs simultaneously generally plausible, which would lead to a sum of all cross-order interface mobilities equal to zero. This is due to the fact that there is no point of origin of the interface coordinates which corresponds to a phase shift of π for all cross-order interface mobility pairs.

On interface systems which are symmetric at all points along the interface, e.g. a circular frame, all cross-order interface mobilities are equal to zero due to symmetry. In such a case, no cross-order interface mobility pair can match all symmetry points with points of origin of the interface coordinates corresponding to a zero phase shift.

In conclusion, the interference process of cross-order interface mobilities for asymmetric structures can be described as non-systematic. The variations in the superposition of all cross-order interface mobilities for different locations of the ideal point excitation, therefore, are presumed to be negligible. For a symmetric interface, the constructive and destructive interference of terms in cross-order interface mobility pairs is found to be highly systematic. Hence, the changes in the sum of all cross-order interface mobilities for different locations of where $s = s_0 = 0$ are expected to be more distinct and therefore cannot be neglected.

4. Influence of cross-order terms for frame-like structures

The interface geometry of built-up structures consisting of beams is predetermined by the structure, see Fig. 1. The transfer path between excitation and response positions for simple beam-frame structures, therefore, matches the interface. In turn, the actual distance that a wave has to travel from excitation to response location equals $|s - s_0|$ or $C - |s - s_0|$. In contrast to plate-like structures [1], the complexity of the interface geometry, therefore, does not affect the influence of the cross-order terms.

The discontinuities associated with the joints or angles of frame-like structures do promote the influence of cross-order interface mobilities. This is due to the fact that in the presence of discontinuities, point and transfer mobilities are sensitive to the location along the interface.

With the knowledge of the physical attributes which promote cross-order interface mobilities, configurations of frame-like structures can be devised where all cross-order terms are equal to zero. Such are closed contour, frame-like structures without discontinuities, which would lead to wave-type conversion or reflections. Theoretically, the geometry of these frames and consequently of the interface can be of any shape as long as the waves do not experience any discontinuity. In terms of interface symmetry, the dynamic characteristics of such structures are identical in both directions along the interface outgoing from any point. Thus, structures without discontinuities are symmetric at all points. Hence, no cross-order interface mobility pair can match all symmetry points with points of origin of the interface coordinates corresponding to a zero phase shift between the two interface mobility terms, see Section 3.

Owing to the primary question addressed, i.e. the influence of the cross-order terms with a uniform force-order distribution, a generic structural configuration was selected for the analytical and experimental work. The system consisting of four homogeneous aluminium beams of 20 mm height and width, forms a rectangular interface, see Fig. 4. The frame-like structure under consideration comprises four points of symmetry, i.e. at points 3, 9, etc. The characteristic dimension of the system for use in the Helmholtz number is chosen to be half the diameter of the rectangle, i.e. L_0. For rectangles with non-extreme aspect ratios, half the diameter is the dimension which is most similar to the radius of a circular interface of equal circumference. Hence, the Helmholtz number characteristics outlined in Ref. [1] can be directly compared with those presented herein.

In order to simulate infinite beams for the experimental setup, the 3 m long beams were embedded in sloped sand at the ends to reduce the reflections. Furthermore, the interface was positioned at the center of the four beams in order to achieve the same symmetry characteristics as with the infinite beams. The frame-like structure was excited with an impact hammer on the upper side, while the accelerometers were fastened at the bottom side. The point and transfer mobilities along the interface were calculated from the force and velocity data.

For the analytical investigation, the frame-like structure under consideration is treated as a built-up structure. Substitution forces and moments at the joints are included for both bending and torsional waves [2]. The point and transfer mobilities of the four infinite beams are obtained by the wave approach employing Euler–Bernoulli theory [3]. A double FFT of the mobility matrix readily yields the interface mobilities.

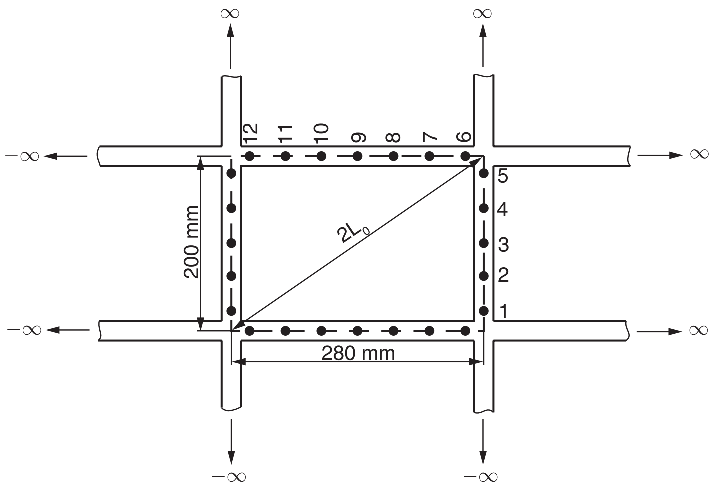

Fig. 4. Rectangular frame-like structure consisting of infinite, homogeneous beams with an interface sampled at $N = 24$ points. – – –, Interface; •, discretization point.

For both the analytical and the experimental analysis, the interface was sampled at 24 equi-distant points, see Fig. 4. With 24 sampling points, the spatial counterpart of the Nyquist criterion allows the determination of the highest Fourier components of order ±12. The validity in frequency is determined by two conditions. The distance between two sampling points has to be smaller than half the governing wavelength [4]. Furthermore, for the Euler–Bernoulli theory to be valid, the height of the beams has to be smaller than six times the governing wavelength. Hence, the results are valid up to approximately $k_B L_0 = 10$.

As pointed out in Section 2, the influence of the cross-order terms on the complex power and the source descriptor and coupling function orders have to be investigated separately. Hence, the present analysis is divided into two subsections, i.e. the influence of cross-order terms with regard to the complex power and subsequently for the source descriptor and coupling function orders.

4.1. Complex power

As pointed out in Section 2, the position of the ideal point excitation has to be varied in order to draw generally valid conclusions, representative of the system under consideration with uniform force-order distributions. For this reason, the two cases of maximum and minimum influence of the cross-order terms are considered in the following.

The strongest influence of the cross-order terms can be expected to occur when the point of origin of the interface coordinates is set to a symmetry point. However, this is not necessarily true, since all possible combinations of cross-order interface mobilities interfere constructively or destructively. Hence, a maximum squares analysis is performed for the deviation between the point mobilities along the interface and the average point mobility, see Eqs. (9) and (10). The point of origin of the strongest influence of the cross-order interface mobilities is found to be point 9, see Fig. 4. The corresponding magnitudes of the various superpositions for the experimental and theoretical analysis are plotted in Fig. 5.

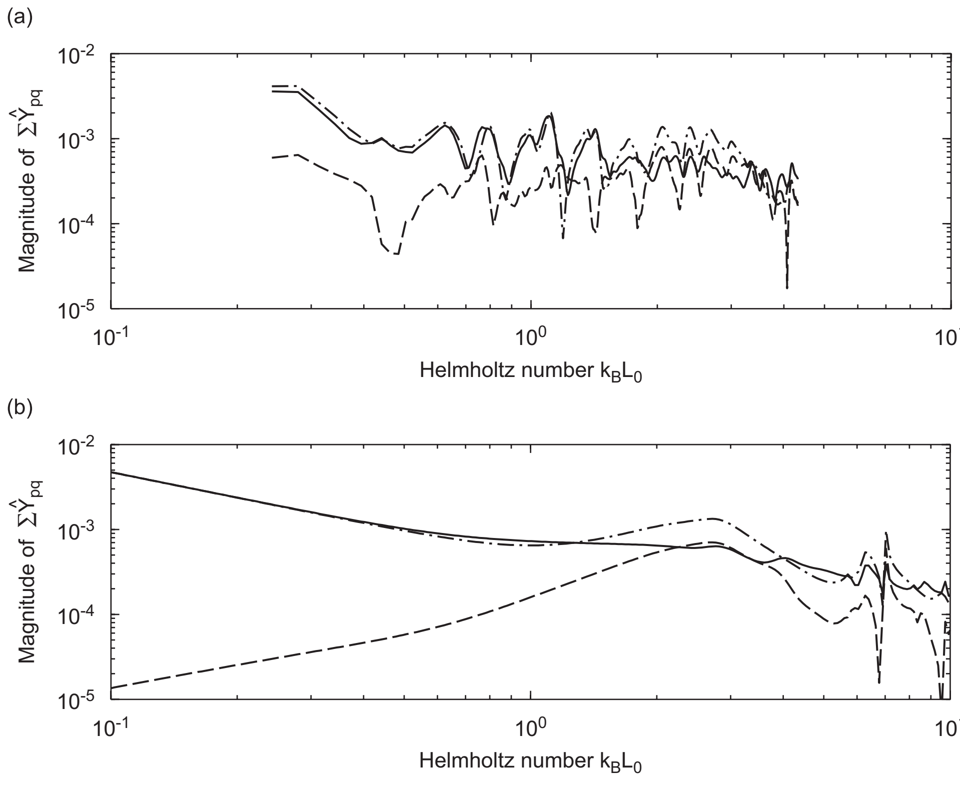

Fig. 5. Superposition of interface mobilities for a rectangular beam frame. The origin of the interface coordinates is located at the point corresponding to the strongest influence of the cross-order terms: (a) experimental analysis; (b) theoretical analysis. —, Equal-order interface mobilities; – – –, cross-order interface mobilities; – · –, all interface mobility terms.

The best fit between the point mobility along the interface and the mean point mobility is found for the position of the ideal point excitation at point 1 for the experimental analysis and at point 7 for the analytical investigation. Thus, the superpositions of the interface mobility terms for the weakest influence of the cross-order terms are presented in Fig. 6.

In Figs. 5 and 6, the quality of the equal-order approximation is indirectly indicated by the magnitude of the cross-order term superposition. Noticeable discrepancies between the true value and the equal-order approximation occur if the cross-order terms are of the same order of magnitude as the sum of all interface mobilities.

For the complex power, assuming a uniform force-order distribution, the influence of the cross-order terms exhibits two distinct frequency regions, see Figs. 5 and 6. At low frequencies, the wavelengths are substantially longer than the characteristic dimensions of the system. Here, excitation and response are similarly dynamically coupled irrespective of their relative locations along the interface. Hence, the matrix of the ordinary mobilities is approximately constant for all combinations of s and s_0. In turn, a dominance of $\hat{Y}_{00}$ is observed for small Helmholtz numbers and the cross-order interface mobilities are of subordinate significance. However, if the in-phase motion of the structure along the interface is constrained at some point, e.g. by means of a support, the cross-order interface mobilities $\hat{Y}_{01}$ and $\hat{Y}_{0-1}$ can be influential. On interface systems with more than two symmetry points, such cross-order interface mobility terms are equal to zero. Hence, for structures with more than two points of symmetry, the cross-order terms are of subordinate influence at low frequencies in general. One may therefore consider the influence of cross-order terms negligible for most interfaces in practice.

At small Helmholtz numbers, the cross-order interface mobilities are of stronger influence in the experimental than in the analytical investigation. In the analytical analysis, the four beams forming the frame-like structure are of infinite length. Without constraints, vibration pattern corresponding to any wavelength are possible. At very low frequencies, therefore, a point excitation will result in an almost constant displacement of the structure along the interface. Hence, the influence of the cross-order terms vanishes with

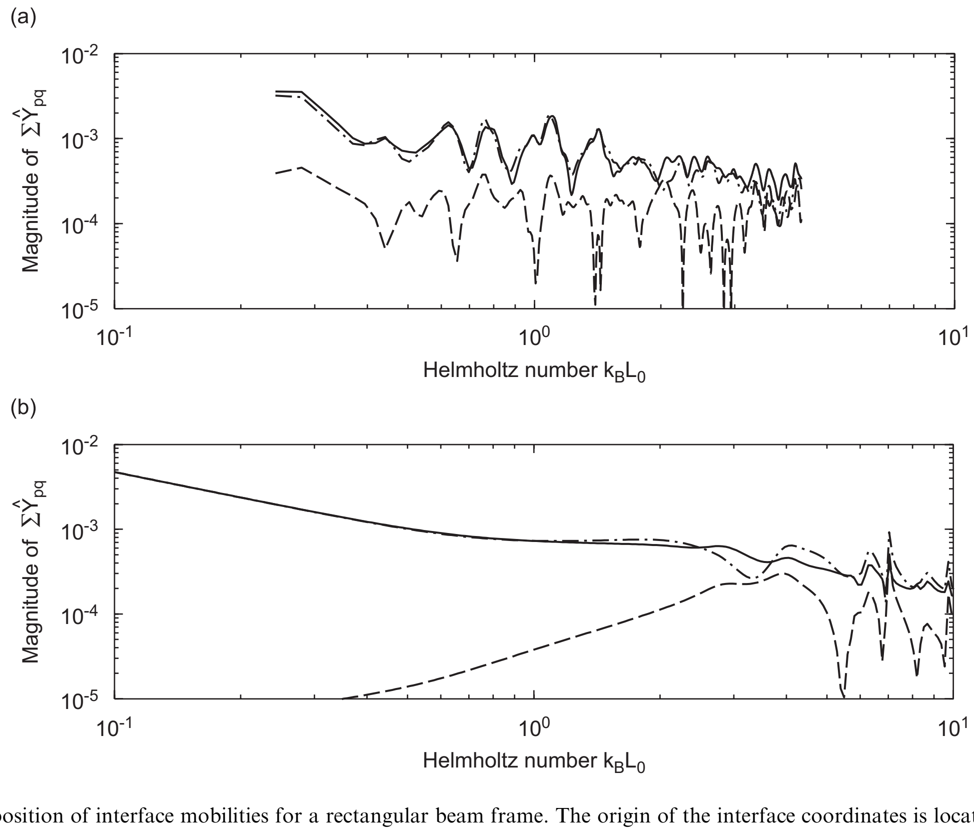

Fig. 6. Superposition of interface mobilities for a rectangular beam frame. The origin of the interface coordinates is located at the point corresponding to the weakest influence of the cross-order terms: (a) experimental analysis; (b) theoretical analysis. —, Equal-order interface mobilities; – – –, cross-order interface mobilities; – · –, all interface mobility terms.

decreasing frequency. In the experimental case, the frame-like structure consists of four finite beams supported by sand at the beam ends. Accordingly, a lowest possible eigenmode is determined by the system. At frequencies below this eigenfrequency, the forced vibration pattern will be similar to that of the lowest eigenmode. The influence of the cross-order terms, therefore, remains approximately constant with decreasing frequency.

In the upper Helmholtz number region, the point and transfer mobilities are sensitive to the location along the interface. This is due to the presence of the structural discontinuities at the corners of the rectangle where the beams are joined. Such joints are associated with wave-type conversion as well as transmission and reflection of impinging waves. Thus, the point and transfer mobility matrices exhibit significant variations along the main diagonal and lines parallel thereto, respectively, see Ref. [1]. In the upper Helmholtz number region, therefore, discrepancies between the true value and the equal-order approximation can occur, see Figs. 5 and 6.

In the experimental rig, the ends of the beams represent additional discontinuities although they are embedded in sand. In contrast to the discontinuities due to the joints, the beam ends are located at a distance from the interface. Hence, the ends of the beams enhance the influence of the cross-order terms at lower frequencies than the corners at which the beams are joined. In Fig. 5(a), the cross-order interface mobilities, therefore, become of the same order of magnitude as the sum of all orders at lower Helmholtz numbers than in Fig. 5(b).

In contrast to plate-like structures, the influence of the cross-order terms on frame-like structures does not decrease with increasing frequencies at large Helmholtz numbers. The structural discontinuities due to the corners or joints are located along the interface. With increasing frequency, these discontinuities therefore do not get further and further away from the interface in terms of wavelengths but remain part of the interface. Additionally, the transmission path between excitation and response positions follows the interface, see Fig. 4. Thus, all transfer mobilities are strongly affected by such discontinuities. On plate-like structures, the transfer mobilities vanish asymptotically with increasing frequency by virtue of divergence. As the wave propagation is one-dimensional in slender beams, the transfer mobilities remain important in the upper frequency region for frame-like structures.

When comparing Figs. 5 and 6, similar characteristics are observed in the lower frequency region. The cross-order terms are of subordinate significance for all possible locations of the origin of the interface coordinates at small Helmholtz numbers. Discrepancies between the equal-order interface mobility superposition and that of all interface mobility terms for maximal and minimal influence of the cross-order terms are mainly observed in the upper frequency region. Here, the superpositions of equal-order and cross-order interface mobilities are seen to be of similar order of magnitude. Although less than one order of magnitude, the variations between the curves of maximum and minimum destructive interference, therefore, have a noticeable effect on the quality of the equal-order approximation.

In Fig. 7, the phase relations of the three groups of interface mobilities are plotted for the points of maximum and minimum destructive interference. The equal-order approximation resembles the superposition of all interface mobilities for small Helmholtz numbers, irrespective of the degree of destructive interference between the cross-order interface mobilities. In the upper Helmholtz number region, the quality of the equal-order approximation is reduced, indicating a stronger influence of the cross-order terms. In this frequency range, the superposition of all equal-order interface mobilities shows different characteristics in Fig. 7(a) and (b). In an overall sense, however, the quality of the equal-order approximation is observed to be independent of where $s = s_0 = 0$.

4.2. Source descriptor and coupling function orders

As pointed out in Section 2, the quality of the equal-order approximation of the source descriptor and coupling function orders can be assessed by analyzing the individual velocity orders. The series of interface mobilities for a few velocity orders are given in Eq. (12) under the assumption of a uniform force-order distribution:

$$
\begin{aligned}
\hat{v}_{-2}/(C\hat{F}) &= \hat{Y}_{-20} + \hat{Y}_{-2-1} + \hat{Y}_{-21} + \hat{Y}_{-2-2} + \hat{Y}_{-22} + \hat{Y}_{-2-3} + \cdots, \\
\hat{v}_{0}/(C\hat{F}) &= \hat{Y}_{00} + \hat{Y}_{0-1} + \hat{Y}_{01} + \hat{Y}_{0-2} + \hat{Y}_{02} + \hat{Y}_{0-3} + \hat{Y}_{03} + \cdots, \\
\hat{v}_{1}/(C\hat{F}) &= \hat{Y}_{10} + \hat{Y}_{1-1} + \hat{Y}_{11} + \hat{Y}_{1-2} + \hat{Y}_{12} + \hat{Y}_{1-3} + \hat{Y}_{13} + \cdots.
\end{aligned}
\tag{12}
$$

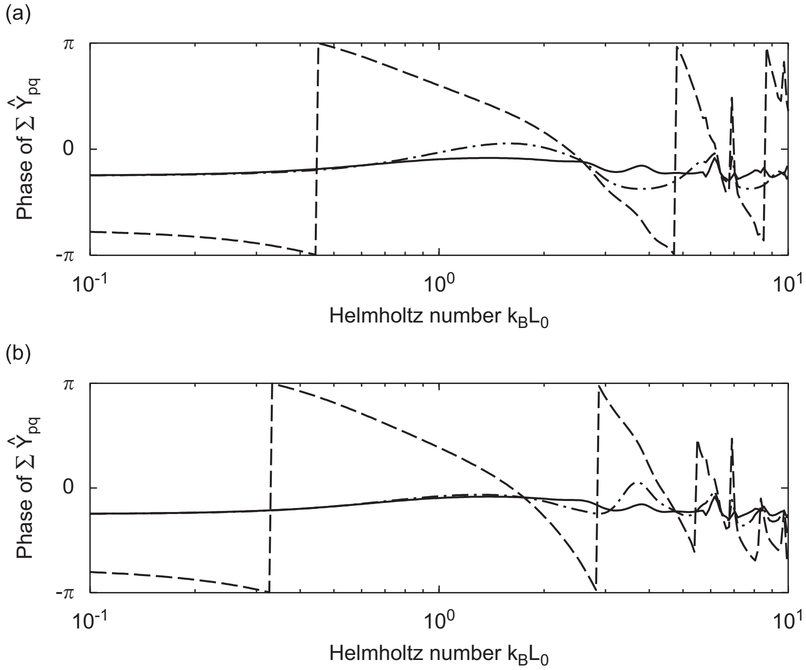

Fig. 7. Analytical results of the superposition of interface mobilities for a rectangular beam frame with the origin of the interface coordinates located at two different positions: (a) point 9; (b) point 7. —, Equal-order interface mobilities; – – –, cross-order interface mobilities; – · –, all interface mobility terms.

The zero-order velocity consists of $\hat{Y}_{00}$ and cross-order interface mobility pairs of type $\hat{Y}_{0q}$ and $\hat{Y}_{0-q}$. Hence, distinct variations can be expected between different positions of the ideal point excitation. All non-zero velocity orders do not include any cross-order interface mobility pairs. Here, the random interference of the cross-order interface mobilities is presumed to result in a less pronounced variation with different locations of the ideal point excitation.

In Figs. 8 and 9, analytical results are presented for a few velocity orders for the beam frame considered. For each velocity order, the origins of the interface coordinates for minimum and maximum destructive interference of the cross-order interface mobilities was determined separately. Again, a least and maximum squares comparison was applied, respectively. The frequency characteristics of the influence of the cross-order terms are found by considering both these cases under the assumption of a uniform force-order distribution.

The quality of the equal-order approximation shows similar frequency characteristics for the zero-order source descriptor and coupling function as well as the complex power. At small Helmholtz numbers, the cross-order interface mobilities have little influence. This feature will change, however, if the structure is asymmetric or exhibits two points of symmetry, see Table 1. In such cases, the interface mobilities $\hat{Y}_{01}$ and $\hat{Y}_{0-1}$ can become important. Larger discrepancies between the zero-order velocity and $\hat{Y}_{00}$ are found in the upper frequency region. Here, the superposition of the respective cross-order interface mobilities is of the same order of magnitude as the equal-order interface mobility of order zero.

For the first-order velocity, the influence of the cross-order terms is seen to be approximately constant with frequency. The magnitude of the series of the associated cross-order interface mobilities is less than one order of magnitude lower than $\hat{Y}_{1-1}$.

For higher orders, the influence of the cross-order terms can be divided into two frequency regions. The transition between these two Helmholtz number regions is set by the maximum in magnitude of the velocity order considered. In the lower frequency region, the equal-order approximation shows a systematic underestimation of the velocity order. In the upper frequency region, the superposition of the cross-order interface mobilities is of the same order of magnitude as the corresponding equal-order interface mobility.

When exciting the frame-like structure with $\hat{F}_0$, the response will include higher-order terms. This is due to the fact that the structure has a lower mobility at the joints than between the joints. Furthermore, the frequency band of the maximum of an interface mobility is related to the order numbers. Interface mobilities of low order numbers are important at low frequencies and those with high order numbers at high frequencies. Hence, interface mobilities which describe the cross coupling from low-order forces are influential at lower

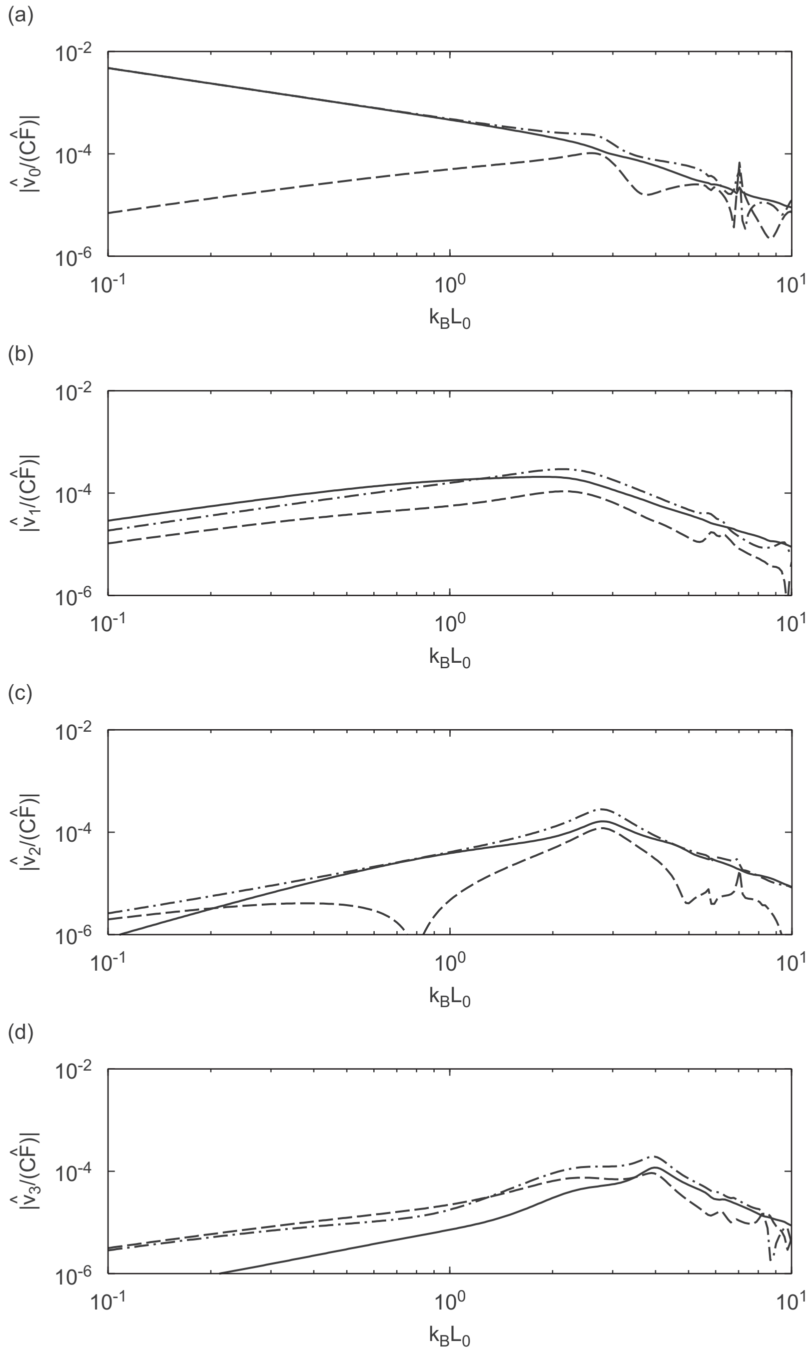

Fig. 8. Analytical results of the interface mobility series for a few velocity orders on a rectangular beam frame. The origin of the interface coordinates is located at the respective points corresponding to the strongest influence of the cross-order terms: (a) $\hat{v}_0$; (b) $\hat{v}_1$; (c) $\hat{v}_2$; (d) $\hat{v}_3$. —, Equal-order interface mobility; – – –, cross-order interface mobilities; – · –, complete interface mobility series.

frequencies than the corresponding equal-order interface mobility. Already a weak cross coupling, therefore, suffices to give an underestimation of high-order velocities using the equal-order terms below the maximum. At small Helmholtz numbers, however, $\hat{v}_0$ exceeds the high-order velocities by several orders of magnitude. Hence, the high-order velocities are of little influence in this frequency region.

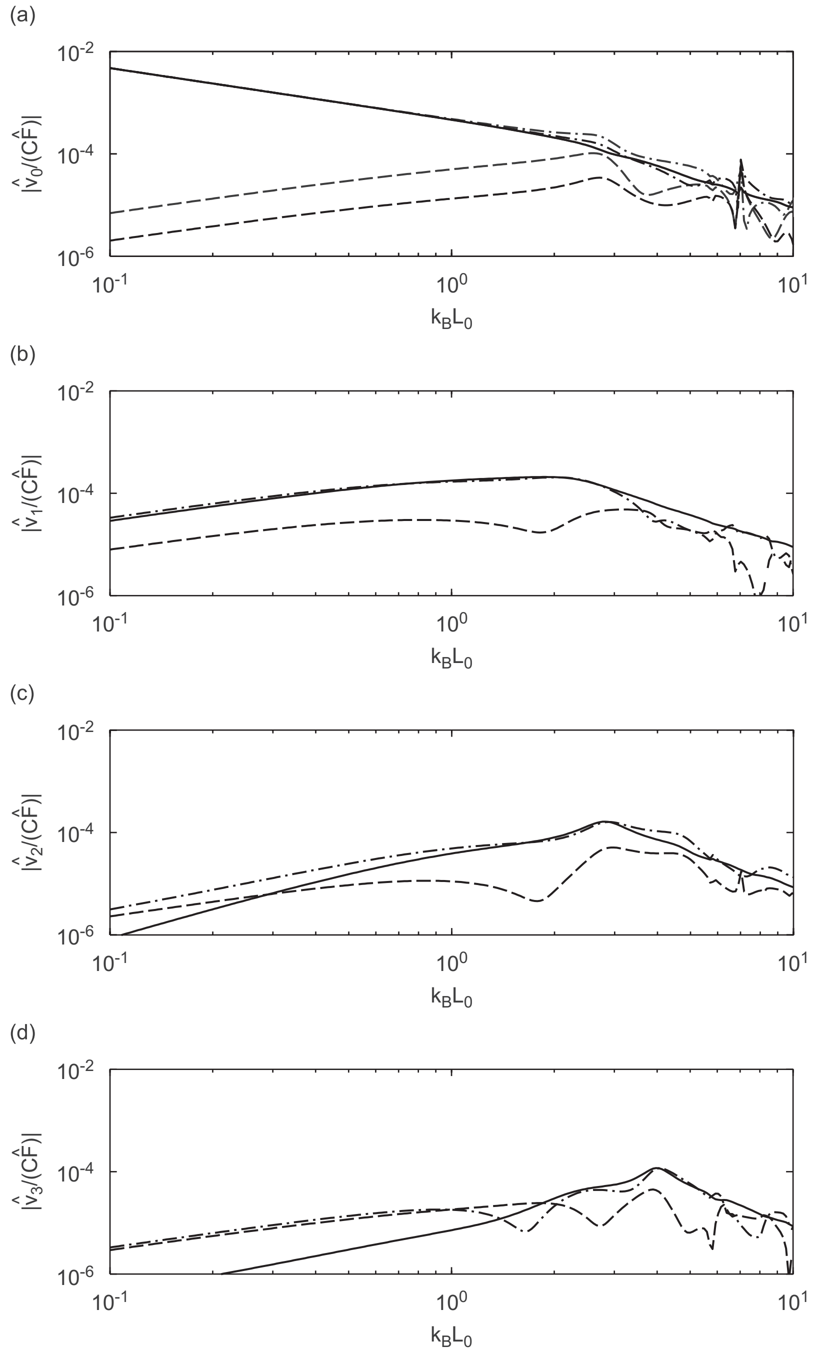

Fig. 9. Analytical results of the interface mobility series for a few velocity orders on a rectangular beam frame. The origin of the interface coordinates is located at the respective points corresponding to the weakest influence of the cross-order terms: (a) $\hat{v}_0$; (b) $\hat{v}_1$; (c) $\hat{v}_2$; (d) $\hat{v}_3$. —, Equal-order interface mobility; – – –, cross-order interface mobilities; – · –, complete interface mobility series.

At frequencies below the maximum of a high-order velocity, the influence of cross-order terms depends on the presence of low-order cross-order interface mobilities. Although not demonstrated, the quality of the equal-order approximation of the first-order velocity can drop significantly at low frequencies, if $\hat{Y}_{10}$ is present. In such a case, the frequency characteristics of the cross-order term influence on $\hat{v}_1$ are similar to those of higher orders.

When comparing Figs. 8 and 9 for $\hat{v}_0$, the same characteristics are observed as for the complex power, see Figs. 5 and 6. This is due to the fact that both the complex power and the zero-order velocity include cross-order interface mobility pairs. For the non-zero velocity orders, only minor differences of the cross-order term influence are observed between Figs. 8 and 9. Without the presence of cross-order interface mobility pairs, the constructive and destructive interference of the cross-order interface mobilities can be described as randomized.

The quality of the equal-order approximation for the phase of the velocity orders shows similar characteristics as of the magnitude. The same applies for the variations of such characteristics for different locations of the ideal point excitation.

5. Concluding remarks

The applicability of the concept of interface mobilities depends on the admissibility of neglecting the cross-order terms. By assuming a uniform force-order distribution, the cross-order terms reduce to the cross-order interface mobilities. For frame-like structures, the cross-order interface mobilities describe the dependence of point and transfer mobilities on the location relative to boundaries and structural discontinuities.

Interface systems for which the dynamic characteristics of the structure are identical in both directions along the interface outgoing from a certain point, can be called symmetric. With the knowledge of the number of symmetry points, it is possible to predict which cross-order interface mobility terms are equal to zero due to symmetry. In turn, information about the influence of the cross-order terms is gained.

The influence of the cross-order terms for frame-like structures on the calculated complex power can be divided into two frequency regions. At low frequencies, the equal-order interface mobility of order zero dominates all other terms. In turn, the influence of the cross-order terms is small. If the in-phase motion of the structure along the interface is obstructed, however, the cross-order terms may become significant. In the upper Helmholtz number region, the sum of all cross-order interface mobilities is seen to be of the same order of magnitude as the superposition of all equal-order interface mobilities. In contrast to plate-like structures, the influence of the cross-order terms does not decrease with increasing frequency for the case of frame-like structures.

For the analysis of the cross-order term influence on the source descriptor and coupling function orders, the series of interface mobilities in the individual velocity orders have to be investigated. The frequency characteristics of the cross-order terms for the zero-order velocity are found to be similar to those of the complex power. For non-zero velocity orders, the presence of low-order cross-order interface mobilities determines the quality of the equal-order approximation. At frequencies above the characteristic maximum of the velocity orders, the cross-order terms are observed to be of the same order of magnitude as the respective equal-order interface mobilities.

The equal-order approximation manages to capture the main trends and overall characteristics of the complex power as well as the source descriptor and coupling function orders. For engineering practice, the omission of the cross-order terms results in an acceptable estimate of both magnitude and phase spectra. However, the present analysis is restricted to the case of a uniform force-order distribution. Hence, the distribution of force orders and their effect on the influence of the cross-order terms remains to be investigated.

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